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Kinematic Point Positioning of a LEO With Simultaneous Reduced-Dynamic Orbit Estimation

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BIOGRAPHIES

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ABSTRACT

A technique for finding, with sub-decimeter precision, the trajectories of Low Earth Orbit (LEO) spacecraft with GPS receivers on board, has been developed at NASA's Goddard Space Flight Center (GSFC). This technique has been tested, by computing a 24-hour orbit for the oceanographic satellite JASON-1. With three independent, precise tracking systems: GPS, DORIS, SLR, and at a height of 1300 km, this satellite currently has the best-determined orbits of any LEO. The GPS data has been post-processed in precise point-positioning mode (with data from a receiver on board the satellite, and precise orbits and clock corrections from an external source). The clock of the receiver has been eliminated by single-differencing between GPS satellites. The resulting trajectory agrees to better than 5 cm (3-D RMS) with "truth": a very precise orbit calculated independently, using well-tested space-geodetic techniques implemented in the GEODYN software used at Goddard SFC.

The method presented here is a form of reduced-dynamic orbit determination that is easily incorporated into pre-existing, precise kinematic software, because it relies on the use of a linearized orbit dynamics theory with a simple analytical formulation, easily programmed in a computer. A purely kinematic trajectory for the rover is obtained simultaneously with the reduced-dynamic orbit. This method is the product of a study aimed at overcoming the limitations of the kinematic approach when dealing with very fast-moving LEOs, by introducing some easily implemented constraints on the solution. Those limitations are a consequence of the short observing times available for estimating the biases in the ionosphere-free carrier-phase combination.

The technique has been tested with a modified version of the kinematic "IT" software developed by the first author. This software already has the option to use the same method, in long-baseline differential solutions, to estimate and correct errors in the GPS broadcast ephemerides, to improve results when these less precise orbits are used.

INTRODUCTION

Precise Point Positioning, LEO Orbit Determination, and the Kinematic Approach. *Precise Point Position (PPP)* is the geolocation of a fixed or moving object, with data from a single GPS receiver, assisted with precise information on the orbits and clocks of the GPS satellites. This contrasts with network-based methods, where the data of several receivers are combined to find the position of one or more of them relative to the others. In that case, precise orbits and clocks are less essential, as they can be estimated simultaneously with the unknown positions of the receivers, or (in the case of the clocks) eliminated as explicit unknowns, by differencing the data (differential GPS). The main advantage of PPP over the network approach is that it is much easier, and faster, to handle, stage and process data from just one receiver, than from many. (Of course, the orbits and clocks are themselves estimated using data from some global network of GPS sites, but the PPP users do not need to do that themselves.)

The main disadvantage, potentially, is the dependence on outside sources for precise information on GPS orbits and clocks. This information, nowadays, is readily available on the Internet by anonymous ftp from the archives of various organizations, so there is no essential difficulty in getting it, as long as one intends to post-process the data. Their quality is easily tested, using them to find the position of a site with coordinates already known and readily available data, such as an International GPS Service (IGS) site. To this date, the experience has been that, for the most part, those orbits and clocks are of a consistently high quality.

The classical approach to precise orbit determination (POD) is the *dynamic* orbit integration method: the main forces acting on the spacecraft are carefully modeled, and the rest is estimated from tracking data, in the form of acceleration parameters (e.g., drag and solar radiation pressure coefficients). This method is irreplaceable if tracking data is sparse and poorly distributed along the orbit. Nowadays, with receivers on low Earth orbiters (LEOs), several simultaneous GPS observations, collected every few seconds, can be used alone or combined with other types of tracking. This has driven the development of a new and very precise technique, known as *reduced-dynamic* [1], [2], where many acceleration parameters are estimated. These are, typically, the successive amplitudes of accelerations in the radial, across-track, and along-track directions that change value every few minutes, in random-walk fashion. Finally, there is the *kinematic* approach, where no dynamics are assumed and the solution depends purely on the geometric strength derived from observing several GPS satellites simultaneously. The idea of using it for LEOs, inspired on the high precision already achieved for aircraft, ships, cars, etc., has been the object of several recent studies.

For the most part, the kinematic results reported for LEOs (e.g., [3], [4]), agree with "truth" to 20-40 cm in 3-D RMS, and a few cm in 3-D Mean. "Truth" consists of (presumably) precise orbits obtained by the dynamic, or the reduced dynamic approach. It must be noted here that these two studies have been made for the satellite CHAMP. Because of this LEO's low altitude, the estimated orbits used as "truth" may not be all that precise. The a priori satellite positions, good to a few meters, are obtained using pseudo-range data only. The a priori constraints on the position of the satellite are either very loose, or non-existent. The reason for the lower precision achieved for LEOs, when compared to terrestrial vehicles, is that those spacecraft move much faster. As they circle the Earth, the GPS satellites visible from them rise and set very often, remaining at useful elevations for only 10 or 20 minutes. With fewer observations collected during those short arcs, the biases in the carrier phase Lc (the ionosphere-free combination of L1 and L2) cannot be estimated well enough to get a very precise kinematic solution. However, the authors of another CHAMP study [5] report an agreement of a few centimeters with "truth". The study is characterized by a rare attempt, with a LEO, to resolve exactly the (double-differenced) carrier phase ambiguities. A large number of ground stations (more than 100) is used.

At the Space Geodesy Branch of NASA Goddard Space Flight Center (GSFC), using the orbit determination and geodetic parameter estimation program GEODYN, different types of tracking data can be combined to estimate precisely the orbits of artificial satellites. With the availability of GPS receivers on board the LEOs of several space missions, one important activity at the Branch is to use their data to obtain precise orbit estimates. At present, the preferred method is to combine, whenever possible, GPS data (double differenced between the LEO and a global network of ground receivers), with other types of tracking data that may be available, such as satellite laser ranging (SLR), Doppler tracking, etc.

A New Approach. The work reported here is the authors' first attempt to use PPP for precise orbit determination. It is also part of a continuing study of kinematic GPS as an ancillary tool for POD work. Previously, they experimented with a hybrid differential technique [6]. The idea was to combine the kinematic and dynamic approaches, by using them consecutively, in separate solutions, fitting dynamic orbits to kinematic trajectories. The LEO trajectories so obtained agreed with "truth" (a precise orbit) to some 15 cm in 3-D RMS, and to a few millimeters in the mean. The procedure was tested with the orbits of JASON-1 and TOPEX.

In the present study, reduce-dynamic and kinematic techniques are employed simultaneously, in a single solution. Analytical orbit perturbation theory is used to put dynamic constraints on the kinematic part of the solution. Moreover, the LEO's position can be calculated at epochs when no adequate GPS observations are available. This is important, because LEO data tend to have gaps and to be prone to various glitches. The unknowns solved for are: carrier-phase biases, instantaneous satellite coordinates, the orbit initial conditions, and a number of acceleration parameters. The LEO's receiver clock is eliminated as an unknown by *single-differencing the GPS data between satellites*. As with other reduced-dynamic procedures, a dynamically integrated a priori orbit, precise to a few meters, is used. This makes it easier to detect cycle slips and other data problems, and allows the use of tighter constraints on the many acceleration parameters to be estimated. (The instantaneous coordinates of the LEO are assigned large a priori uncertainties—100m each). A less precise form of point positioning, with pseudo-range only, that needs no preliminary LEO orbit, is used to start the process from scratch. With simultaneously received signals from four or more GPS satellites whose positions and clock errors are known, the LEO's coordinates and receiver clock error are obtained by solving a system of quadratic equations to find the intersection of their space-time light-cones. A dynamic orbit is fitted to this solution, to get the a priori orbit.

The JASON-1 Oceanographic Satellite. Satellite radar altimetry is a form of remote sensing from space where highly precise radar altimeters on spacecraft are used, primarily, to map the irregular shape of the mean sea surface, and monitor its changes world-wide. This gives valuable information on currents, on anomalies in the gravity field that hint at the structure of the Earth's crust and interior, on changes in sea level associated with climate change, and on interactions between ocean and atmosphere.

The interpretation of radar altimeter data requires a very good knowledge of its position in space. Therefore, precise orbit determination is a problem of the greatest importance for altimeter missions. Of those with which the Space Geodesy Branch at GSFC has been directly involved, the US-French mission TOPEX/Poseidon was the first one where a space-qualified dual-frequency GPS receiver was used, experimentally, as a precise tracking device.

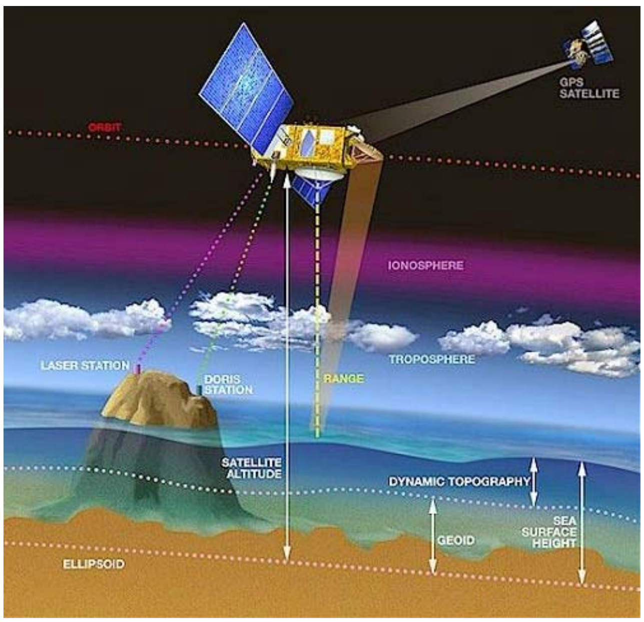

Figure 1. The JASON-1 oceanographic mission. This figure shows the LEO and also illustrates the mission concept. The large Earth-pointing radar altimeter dish is at the bottom. On top and at the front are two GPS antennae. The microwave radiometer slanted antenna dish is seen in front section. The radiometer is used to correct the altimetry for refraction. JASON-1 also carries SLR laser retro-reflectors and a DORIS Doppler receiver (small antenna pointing downwards, behind the altimeter's dish).

This US-French cooperation continues with JASON-1, shown in Figure 1. This is another oceanographic mission with a satellite that carries radar altimeters among its sensors, and, the same as TOPEX, is tracked with GPS on board, satellite laser ranging (SLR), and the French Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS) system. JASON-1 has been placed on the same orbit as TOPEX/Poseidon, which is still in operation. Until recently, one followed the other above the same ground-track (Figure 2), passing over the same point on Earth about one minute apart. The near-circular common orbit has a mean height of 1336 km, to reduce the effect of air drag, while keeping below the inner van Allen band, a world-encircling torus of trapped energetic radiation. The inclination is 66 degrees, providing global coverage of all main ice-free bodies of water. The ground-track is repeated almost exactly every 9.9 days. Proximity to the radiation band, and repeated passes over the South Atlantic Magnetic Anomaly (a large area of weaker field, now centered over southern Brazil, where the van Allen band reaches its lowest altitude), as well as software glitches, may be causing data gaps and other problems [15]. It also should be noted that the GPS 1 and 2 antennae on board are both canted 30^{o} off vertical.

The very high orbit (for a LEO) was originally chosen to make sure that it could be estimated with the greatest precision without GPS, which was experimental in TOPEX/Poseidon, and could be used for POD only until Anti Spoofing begun, in early 1994. With a modern GPS receiver on board, and the same orbit as TOPEX, at present, JASON-1 is the LEO with the best determined orbits, which are, therefore, ideal for use as "truth" when testing new positioning techniques.

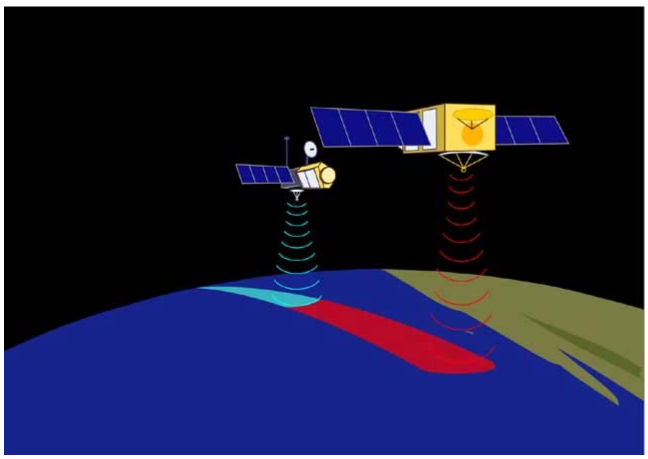

Figure 2. TOPEX/Poseidon and JASON-1 (nearest one in this picture), in their present co-orbiting configuration. The mean height of the surface above the reference ellipsoid is measured along a swath the width of the radar altimeter "footprint" (several km), by timing the return time of the radar pulses, and by precisely determining the orbit of the spacecraft, to know its height above the ellipsoid.

THEORY

Kinematic and Reduced-Dynamic Methods. *Kinematic* GPS is a purely geometric technique: using receiver data from all simultaneously available GPS satellites to estimate the differences (dx, dy, dz) between the instantaneous a priori and actual coordinates of an object. This is done by forming observation equations linearized about the a priori positions of the vehicle, then solving them to find dx, dy, dz by a least-squares fit to the data and, finally, using the result to correct the a priori positions. In terms of their dynamics, dx, dy, dz are treated as white noise, or zero-memory processes. This means that the position estimate at one epoch is not determined by previous ones, nor does it determine future ones. The transition matrix is a 3x3 null matrix. In this study, the a priori uncertainty, or system noise is 100 m per coordinate (one sigma). On the other hand, in the *reduced-dynamic* approach, the corrections dx, dy, dz to the a priori LEO coordinates, along the whole orbit, are the solutions of differential equations. These are derived from the equations of motion for the satellite, based on Newton's laws. Although the gravitational force, air drag, etc., are non-linear functions of position or velocity, the equations have to be linearized in order to implement a least-squares estimation procedure, such as a recursive Kalman filter-smoother. The linearized equations are known as the *variational* equations, because their solutions, to first order, are the changes in the orbit that would be caused by small variations in the values of the unknowns. These unknowns are the corrections to the initial position (dx0, dy0, dz0), velocity (dx0', dy0', dz0'); and also a series of corrections ($da_x(t_k)$, $da_y(t_k)$, $da_z(t_k)$, where t = time) to the accelerations acting on the satellite in the directions x, y, and z, as calculated when integrating the a priori orbit. For this study, the initial dynamic position and velocity components dx0, dx0', etc., are assigned a priori uncertainties of 1 m and 10^{-3} m/s, respectively, corresponding, roughly, in size, to the errors in the a priori orbit. The unknown accelerations da_x, etc., are treated as piece-wise constant random walks that change at successive epochs t_k, (k = 1, 2, 3,.4,...), which coincide with some of the filter updates. Their system noise is of the order of 10^{-8} m/s^2. They are assumed to remain constant over intervals much shorter than the orbit period. For example, they may be allowed to change every 20 minutes or so for GPS satellites, and every 2 minutes for LEOs. In this work, the values of the orbit unknowns are considered to be, in themselves, of little interest. They are useful for constraining the kinematic solution, and also for calculating orbit positions at all epochs of interest, even those with no useful GPS data.

Constraint Equations. The constraints put on the kinematic solution dx, dy, dz take the form of three (pseudo) observation equations where all data equal zero:

$$0+vx = dx - f_x(dx0, dy0, dz0, dx0', dy0', dz0', t) + fa_x(da_x, da_y, da_z, t) \tag{1a}$$

$$0+vy = dy - f_y(dx0, dy0, dz0, dx0', dy0', dz0', t) + fa_y(da_x, da_y, da_z, t) \tag{1b}$$

$$0+vz = dz - f_z(dx0, dy0, dz0, dx0', dy0', dz0', t) + fa_z(da_x, da_y, da_z, t) \tag{1c}$$

The "noise" terms "v" are purely formal: their variances cannot be zero, because of the type of Kalman filter used, and also allow some "slack" between kinematic and dynamic solutions (they are set here to 1 cm of "noise" per pseudo-observation). Functions fx, fy, fz represent the effect of the initial state errors, and fa_x, fa_y, fa_z, that of the acceleration errors (here and in what follows, the "t_k" are omitted, for convenience). The former terms are the *homogeneous* solution, and the latter are the *forced* solution, of the variational equations. Their sum is the *complete* solution. Because the constraint equations (1a-c) force the looser kinematic and stiffer dynamic solutions to agree at certain epochs (although not exactly), all unknowns in the real observation equations, the Lc biases and the kinematic dx, dy, dz, are made to converge faster.

Analytical Orbit Dynamics. For orbits that depart only slightly from a circle, the variational equations have a simple form in a special reference frame. In this frame, the x axis is perpendicular to the orbit plane, the z axis is on the line from the Earth's center to the satellite's, pointing away from Earth. The y axis, approximately aligned with the velocity vector, is perpendicular to the other two, and oriented so as to complete a right-handed triad. The origin is at the Earth's center of mass, and the frame rotates, to keep pointing to the satellite's center of mass. The axes x, y, z are also called, respectively, across-track, or out of plane, along-track, or traverse, and radial axes. The variational equations in this frame are:

$$dx'' = da_x - n_o^2\, dx \tag{2a}$$

$$dy'' = da_y - 2n_o\, dy' \tag{2b}$$

$$dz'' = da_z + 3n_o^2\, dz + 2n_o\, dy' \tag{2c}$$

Where "n_o" is the orbit angular frequency (~10^{-3} radian/s). These expressions, known as Hill's equations, have simple analytical solutions, easily programmed in a computer, that are very useful both for understanding the behavior of satellite orbits, and for estimating orbit errors [7], [8].

The solutions of interest are:

$$f_x = dx0\,[\cos n_o t] + dx0'\,[\sin n_o t/n_o] \tag{3a}$$

$$f_y = dy0 + dy0'\,[4/n_o \sin n_o t - 3t] + dz0\,[6\,(\sin n_o t - n_o t)] + dz0'\,[2/n_o\,(\cos n_o t - 1)] \tag{3b}$$

$$f_z = dz0\,[-3 \cos n_o t + 4] + dz0'\,[1/n_o \sin n_o t] + dy0'\,[2/n_o\,(1 - \cos n_o t)] \tag{3c}$$

And, for da_x, da_y, da_z, constant between epochs t_k and t, with $t_g = t\text{-}t_k$:

$$fa_x = da_x \, [1/n_o^2 \, (1 - \cos n_o (t_g)] \qquad (4a)$$
$$fa_y = da_y \, [4/n_o^2 \, (1 - \cos n_o tg) - 3/2 \, t_g^2] + da_z \, [2/n_o^2 \, (\sin n_o t_g - n_o t_g)] \qquad (4b)$$
$$fa_z = da_z \, [1/n_o^2 \, (1 - \cos n_o t_g)] + da_y \, [2/n_o^2 \, (n_o t_g - \sin n_o t_g)] \qquad (4c)$$

Kinematic solutions are usually made in Earth-fixed coordinates, so the constraint equations (1a-c) have to be transformed using the instantaneous rotation matrix between the frame of (2a-c) and the Earth-fixed frame.

Filter Updates. The expressions for f_x, f_y, f_z, together with their time derivatives, are all that is needed to form the partition of the state transition matrix corresponding to the states of the satellite. The elements of this matrix are the square brackets (and their derivatives) in (3a-c), arranged in six columns and rows, one for each state. In what is known as the *pseudo-epoch state* approach [9], the initial state variables are actually allowed to change with time, so that, at the epoch of any filter update, the homogeneous solution of the equations (the 6-vector with components f_x, f_y, f_z,and f_x', f_y', f_z') always equals the complete solution (including the effect of all accelerations $da_x(t_k)$, da_y, da_z). This simplifies considerably the filter update, because (by choosing all other states to be constants, random walks, or white noise), the overall state transition matrix is a diagonal matrix with only zeros and ones on the main diagonal. The stochastic update at $t = t_{k+1}$ of the pseudo-epoch state consists of adding a 6x6 system noise sub-matrix Q to the 6x6 partition of the filter covariance matrix corresponding to the pseudo-epoch states of a given satellite. Each such sub-matrix is given by the expression:

$$Q = \Phi(t_{k+1} - t_0)^{-1} \, F \, q_a \, \{F \, \Phi(t_{k+1} - t_0)^{-1}\}^T \qquad (5)$$

Where: Φ is the 6x6 satellite state transition sub-matrix from the initial epoch t_0 to t_{k+1}, so its inverse represents the backward transition from t_{k+1} to t_0; "A^T" means "the transpose of A"; F is a 6x3 matrix whose elements are the time integrals, in the interval t_k to t_{k+1} , of the square brackets in the expressions (4a-c) of fa_x, fa_y, fa_z; q_a is the diagonal covariance matrix of the unknown da_x, da_y, da_z. a priori values (the system noise).

Further Considerations. When adjusting the GPS orbits, (possible in network mode, with more than one receiver), the same theory can be used to estimate errors in the orbits of the GPS satellites. The orbit pseudo-epoch state components then appear as unknowns in the observation equations for the carrier-phase and pseudo-range double differences.

To obtain the satellite position at any epoch t, the smoother estimate nearest to t is used to calculate fx(t), fy(t), fx(t), and these are then added to the x, y, z of the a priori, dynamically determined orbit. It does not matter if there are no GPS data available at that epoch. For the LEO solutions reported here, the best results have been obtained by zeroing out the coefficient for dy0 in equation (3b), and all the coefficients in equation (4b). This effectively fixes dy0 and da_y to zero (since all the unknowns have zero a priori values).

TESTING THE TECHNIQUE

Outline of the Procedure. These are the main steps:
(a) Employing precise GPS clock corrections and orbits (used also in (c) and (d)), a preliminary determination of the position and clock error of the LEO is made, using only the pseudo-range data from its GPS receiver. This means solving, at each epoch, a set of n quadratic equations (one for each simultaneous pseudo-range measurement) with four unknowns, to find the intersection of n 4-dimensional light-cones with their vertices at each of the n GPS satellites in view. In this way, the orbit determination can be started without previous knowledge of the LEO's orbit.
(b) A preliminary orbit is dynamically fitted to the pseudo-range solution.
(c) The preliminary dynamic orbit from (b) is used to help find and correct cycle slips in the carrier phase data single-differenced between GPS satellites. Movement along the orbit is smooth, so sudden jumps in the ionosphere-free combination may reveal cycle slips.
(d) A combined kinematic and reduced-dynamic solution is made using the carrier phase corrected for cycle slips in (e), with the orbit from (b) as a priori trajectory.
(e) The results from the reduced-dynamic part of the solution (satellite states and acceleration parameters) are used to calculate corrections to the a priori orbit. The corrected orbit is then checked by:
(1) Comparing it to a very precise orbit estimate (POE), based on JASON-1's GPS and SLR tracking.
(2) Computing distances from the corrected orbit to some laser-ranging sites, and comparing these distances to corresponding laser ranges.

The Initial Dynamic Orbit. GEODYN, the main geodesy and geodynamics analysis software at Goddard Space Flight Center, was used to fit an orbit to the pseudo-range-only preliminary solution. The forces acting on the satellite were modeled with a box-wing model for the effect of solar radiation pressure and drag, and with the JGM3 gravity field model [14]. The unknowns solved for were the orbit six initial state components, as well as a few force-related parameters (classical dynamic approach). Those were: one drag coefficient every four hours (TOPEX) or eight hours (JASON), and one daily set of four or five acceleration parameters (along- and across-track amplitudes of the sine and cosine of the mean anomaly, and also a small constant acceleration across track in the JASON solutions). These unknowns represent the lumped effect of small forces not modeled, or modeled incorrectly [7, ibid.] As explained below, a more precise technique has been used to obtain the "truth" orbit.

The "Truth" Orbit. To verify the quality of the results, a very precise orbit for JASON-1, recently estimated for a different project, also with GEODYN, has been used as "truth". For nearly a decade, starting with the TOPEX/Poseidon mission, very precise orbits for altimeter satellites have been computed at the Space Geodesy Branch of NASA's Goddard Space Flight Center. Considerable efforts have been made to validate the software and the algorithms, and to study the orbit error characteristics [10], [2]. A number of procedures have been used and refined to assess the quality of the results [2]. These include the analysis of orbit overlap differences, post-fit data residuals, and the fit to SLR and other data deliberately not used to estimate the orbit. They also include comparisons with orbits produced by other institutions, and between data sub-set solutions (e.g. orbits determined only with SLR data, compared to DORIS-only solutions), and altimeter crossover residual analyses. Most recently, highly accurate ephemerides for over a year of 9.9-day repeat cycles of the orbit of JASON-1 have been computed at GSFC [2, ibid.], with nearly continuous data available from the on-board dual-frequency BlackJack GPS receiver [15]. With combined GPS and SLR tracking, and the reduced-dynamic method, precise orbits have been calculated in consecutive 30-hour arcs, each overlapping by 3 hours with both the previous and the following arc. Tested in the various ways already enumerated, these orbits have been found to have radial accuracy at the 1 cm level [2, ibid.]. These solutions have been found to be consistent (in arc overlaps) at the 4 mm level radially and 13 mm level 3D [2, ibid.]

GPS Software. The results shown here were obtained by sequential processing with a Kalman filter and a smoother. The first author's "IT" GPS software, modified for this study, already allowed the simultaneous determination of a variety of unknown parameters, in differential, PPP, static and kinematic modes [11], [12], [13]. The parameters include: kinematic corrections to the vehicle a priori position (treated as three "white noise" states, with a 100 m a priori one-sigma uncertainty per coordinate); ionosphere-free combination biases (treated as constants, each with a 10m a priori sigma). Also, although not applicable to LEOs: errors in the tropospheric corrections made for each site (estimating the wet zenith delay and two horizontal gradients); GPS satellite orbit errors; and error in reference station coordinates. The newly added LEO states were assigned the uncertainties mentioned in the previous section. Both the initial and "truth" JASON-1 orbits were converted into GPS antenna-center trajectories, and vice-versa, using the satellite center of mass/antenna center offset, and spacecraft orientation information (available in quaternion form). The GPS satellite clock corrections and precise orbits were chosen from the same solution, so their errors would tend to cancel each other. Corrections were made for relativistic clock variations with height, and for transmitter and receiver antenna offsets in the GPS satellites and in JASON-1. Only the GPS transmitter antenna phase windup was corrected. No corrections were made for antenna pattern (or phase center variations)

EXAMPLE

Alternative Notation. In what follows, the rotating dx, dy, dz defined on the Theory section, will be designated dC, dL and dR, respectively (for Cross-track, aLong-track, and Radial), to make clear they are not in an Earth-fixed frame.

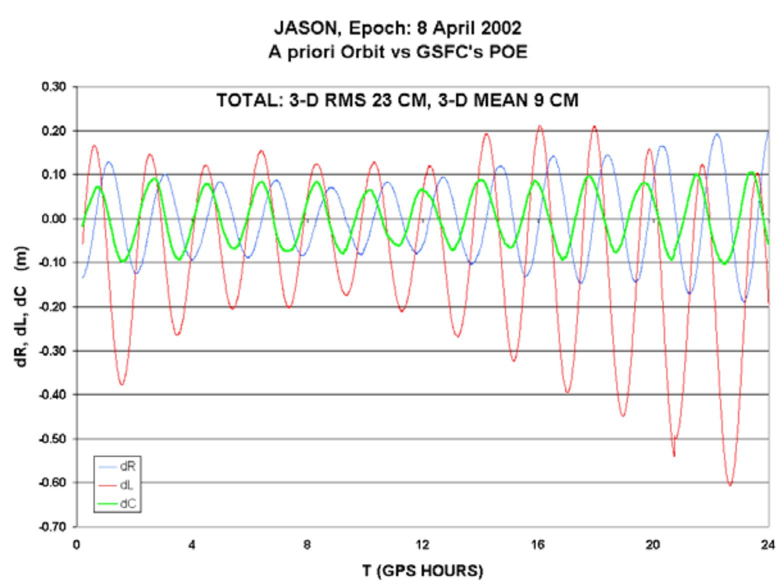

Figure 3. Step (c): Differences between a priori dynamic orbit and "truth".

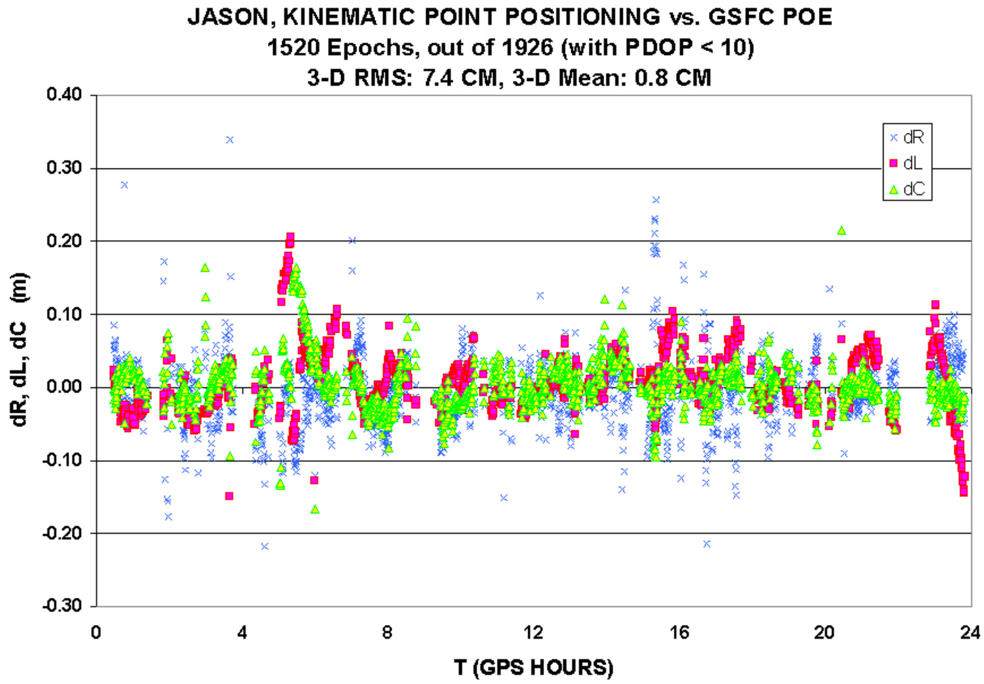

Figure 4. Differences between the kinematic part of the solution, and “truth”.

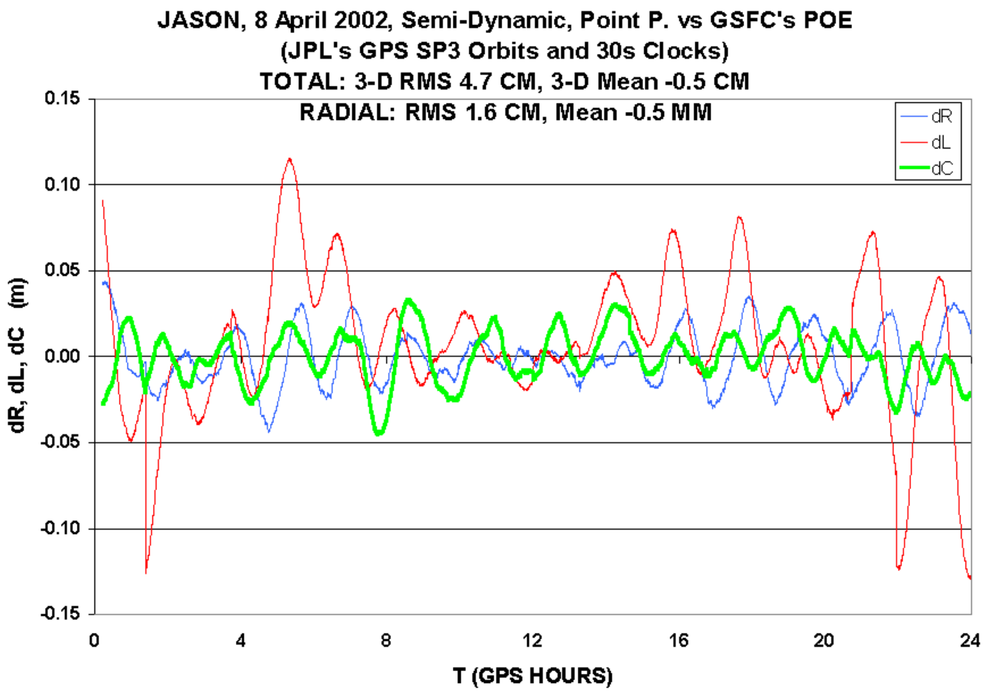

Figure 5. Difference between the reduced-dynamic part of the solution, and “truth”.

The Test Solution. The results of a 24-hour solution for April 8, 2002 are shown in Figures 3-5 above. Figure 3 is a plot of the differences between the a priori orbit (dynamically fitted to a preliminary pseudo-range only solution), and Goddard’s precise orbit estimate (POE), used as “truth”. The a priori orbit differs from “truth” by as much as 60 cm in the along-track direction, and as much as 10 and 20 cm in the across-track and radial directions.

Figure 4 shows the differences between the kinematic part of the solution and "truth". Because the a priori orbit was computed at 30-second intervals, the Precise Point-Positioning solution (both kinematic and reduced-dynamic parts) was also made at 30-second intervals, although the JASON-1 receiver provides data every 10 seconds. JPL's 30-second GPS clock corrections and precise orbits were obtained from the CDDIS on-line archive at Goddard.

There were several long breaks in the data, and many epochs with poor geometry (very large PDOP values). This was due, partly, to few satellites being in view above the elevation cutoff of 6^o chosen by trial and error (a problem made worse by the antennae in JASON-1 not being mounted horizontally), and partly to many data being edited out for bad reception (low signal to noise ratio). In consequence, the total number of 30-second points in the computed 24-hour kinematic trajectory was reduced to 1760, from a possible 2880. The differences plotted in the figure are only those at epochs when the PDOP was less than 10, which further brings down their number by 240, to the 1520 epochs shown in the plot.

Figure 5 shows the plot of the differences between the reduced-dynamic part of the solution and "truth". The ringing pattern discernible in the plot is typical of the difference between two very close orbits, and reveals the resonant nature of small orbit perturbations, implicit in equations (2a-c). Each oscillation has the same period as the satellite orbit, close to two hours. As explained earlier, this part of the solution was easily interpolated to any epochs of the a priori orbit, including those with not enough GPS data to compute points for the kinematic part. Table 1 lists statistics of the differences between each solution and the precise orbit, or "truth". The results for height (dR) are also listed, because this component is the most important in altimeter missions such as JASON-1. Finally, the results of testing the reduced-dynamic part, with laser ranging data of six International Laser Ranging Service (ILRS) sites from around the world, are listed in Table 2. The laser ranges (normal points) were first corrected for site-dependent biases, then had subtracted the computed distances between the estimated positions of JASON-1 and the sites. The RMS of the differences is an independent measure of the accuracy of the solution.

Discussion. In general, the results listed here show that both parts of the solution are substantially better than the a priori orbit. Further work might lead to better choices of a priori uncertainties for the reduced-dynamic unknowns and of the degree of coupling between kinematic and reduced-dynamic trajectories (determined by the level of "noise" introduced in the constraint equations (1a-c)). It may also lead to a better understanding of the strengths and limitations of the new approach. One useful application could be the use of the trajectories so obtained, to validate POD work done with more conventional techniques.

TABLE 1
Departure of the Solutions from "Truth"
(Centimeters)

TRAJECTORY	3-D RSS	dR (RMS)	dR MEAN	Number of Points
A Priori Orbit	23.0	8.8	0.02	2880
Kinematic	7.4	4.9	-0.6	1520
Reduced-Dynamic	4.7	1.6	-0.05	2880

TABLE 2
Agreement between the Reduced-Dynamic Part of the Solution and Independent SLR Tracking
(RMS in Centimeters)

ILRS SITE	RMS OF FIT	No. of Points
WETL	2.0	44
RIGL	4.2	70
METL	3.5	22
HERL	2.3	75
MONL	1.3	20
YARL	4.7	9

CONCLUSIONS

The procedure described here has been tested in a 24-hour solution for the satellite JASON-1, a LEO chosen because its orbit is probably the best determined, at present. Using the new method, both kinematic and reduced-dynamic trajectories have been obtained simultaneously. They differ from a very precise "truth" orbit by less than 10 cm, 3-D RMS, with sub-centimeter mean differences. In particular, the reduced-dynamic part of the solution is within 5 cm (3-D RMS), and 0.8 mm (3D-mean) of "truth", while the fit to independent laser ranging data is about 3 cm RMS.

Applying antenna pattern and phase windup corrections to the JASON-1 GPS data, could reduced those differences further. So could some better tuning of the values of the priori uncertainties assigned to the reduced-dynamic unknowns, and to the constraint equations (1a-c). More tests, using GPS data from other satellites, should help clarify and develop these ideas further.

The results, so far, show that with the proposed procedure one can obtain precise trajectories for LEOs such as JASON-1, probably good enough to be used, among other things, in new types of validation tests for orbits estimated with other POD procedures.

With the approach described here, relatively small modifications are needed to add precise orbit determination capabilities to some pre-existing kinematic software.

Because of the simplicity of handling data from a single receiver, and the faster turnaround time that this makes possible, Precise Point Positioning, however implemented, seems destined to have many practical space applications.

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